Open Quantum Systems for Quarkonia

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HEP Seminar
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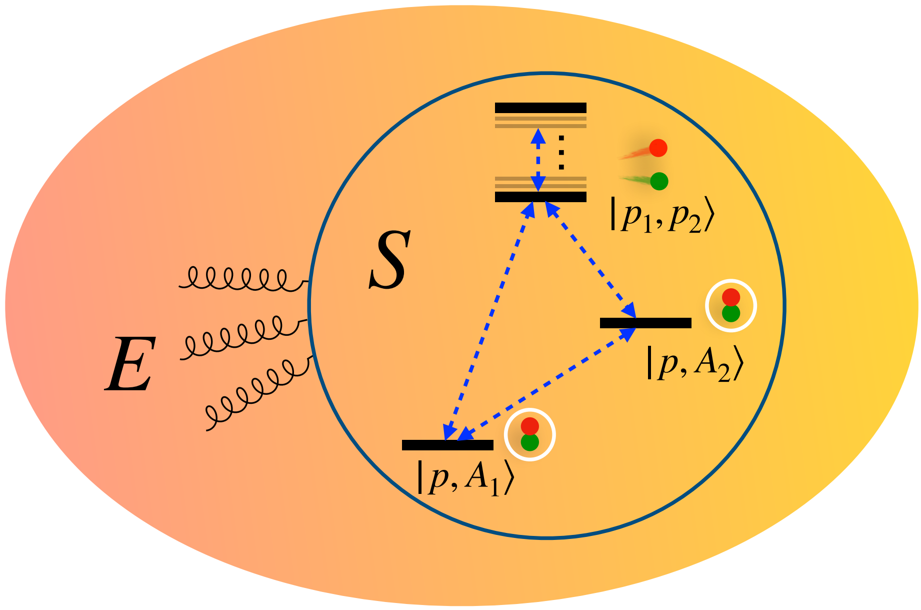
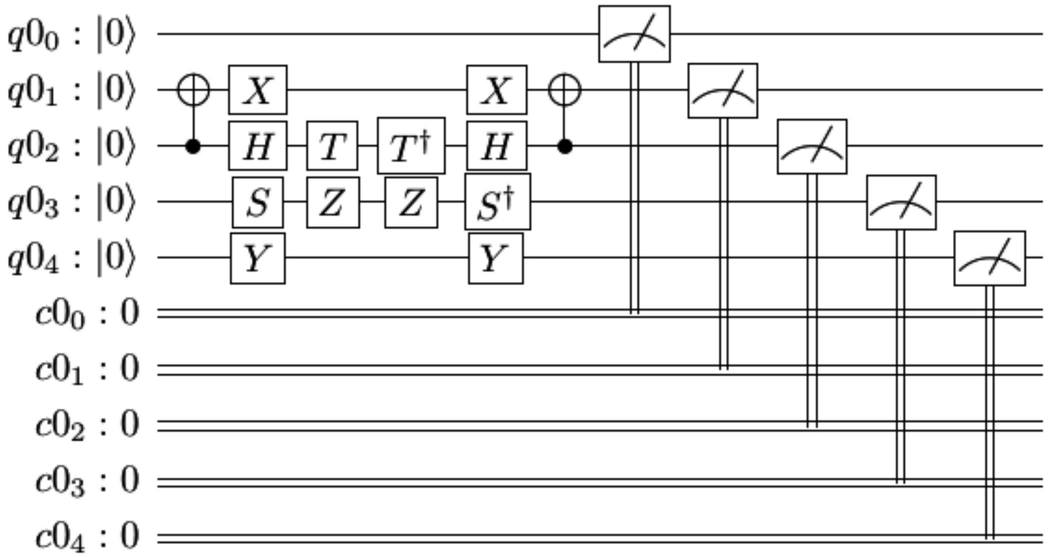




Contents

- Introduction: quark-gluon plasma (QGP) and heavy ion collision, hard probes of QGP: heavy quarks (bound states)

- Open quantum system framework for heavy quarks (quarkonia), effective field theory, Lindblad equation, connection with semiclassical transport

- Quantum simulation of open quantum systems

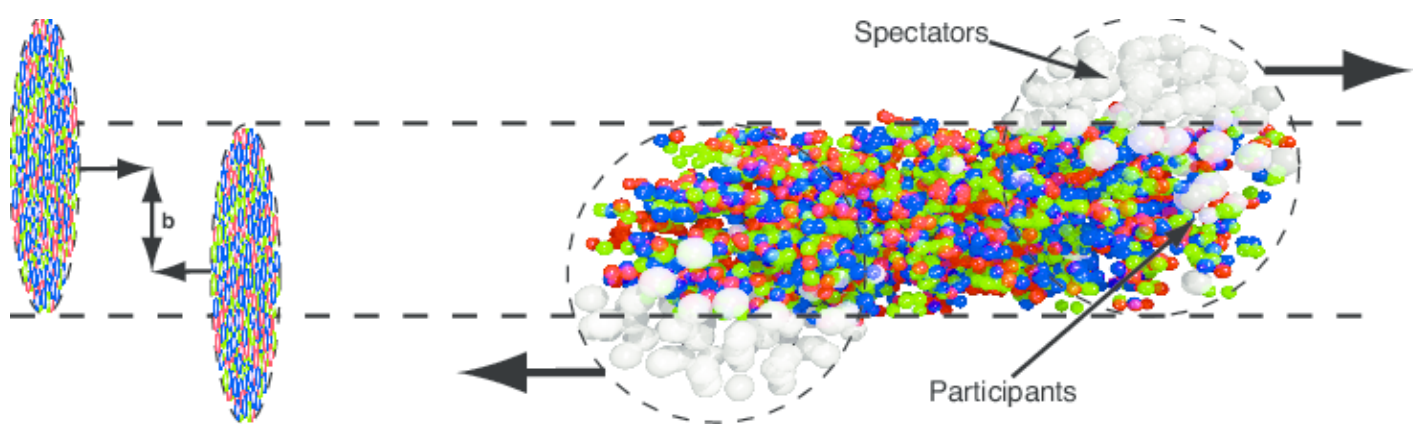
    - Two-level system

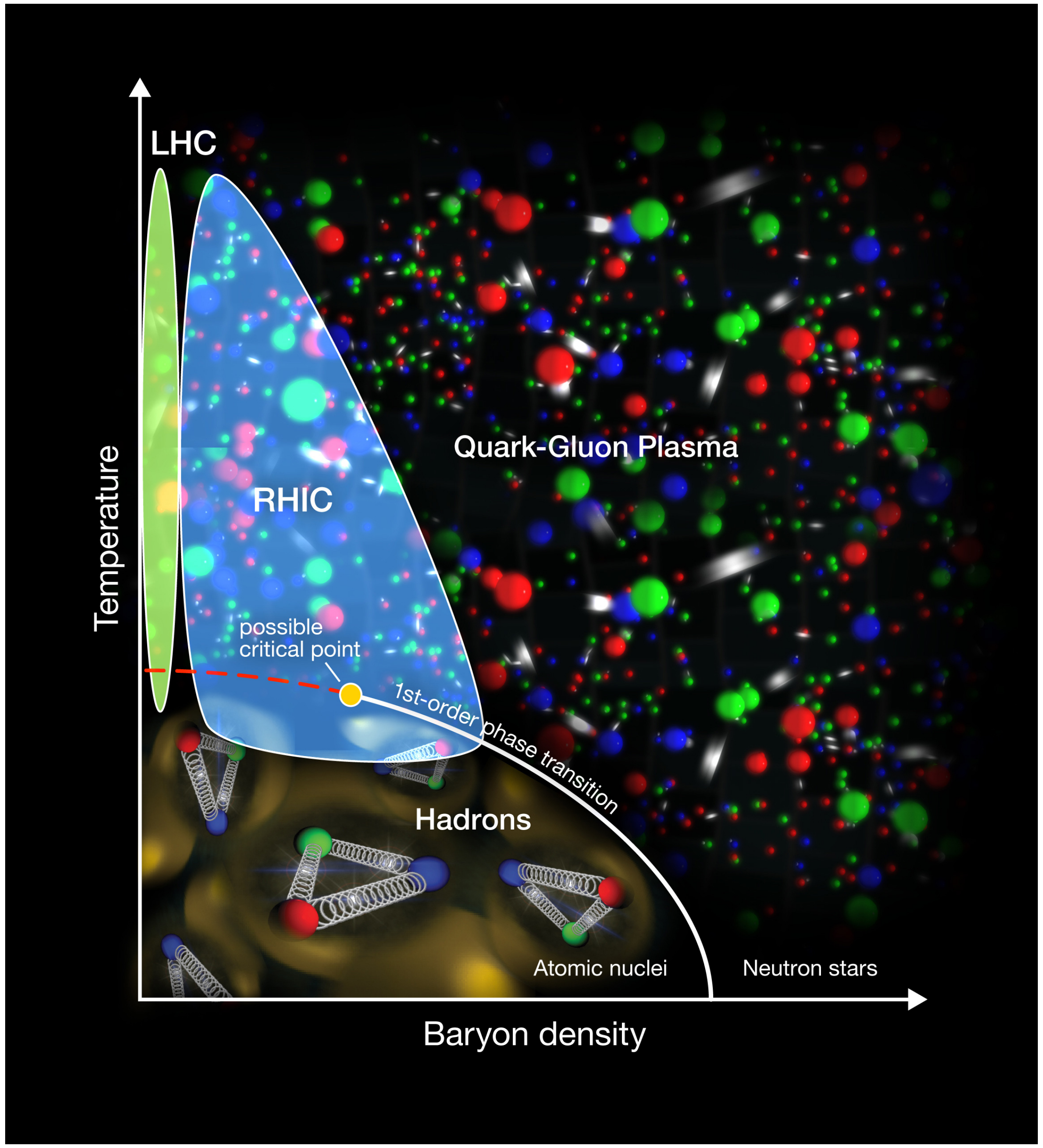
    - U(1) gauge theory in 1+1D (Schwinger model)

Quark-Gluon Plasma and Heavy Ion Collision

- Asymptotic freedom —> deconfined phase of QCD matter expected at high temperature / density —> QGP

- Study QGP: heavy ion collision experiments at RHIC and LHC





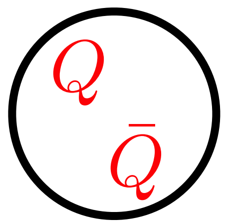
- QGP fireball: strongly coupled, lifetime ~10 fm/c, temperature 150—600 MeV

- Hard probes of QGP: large energy scale, heavy quarks, quarkonia and jets

I. Heavy Quarkonia as Open Quantum Systems

Quarkonia inside QGP

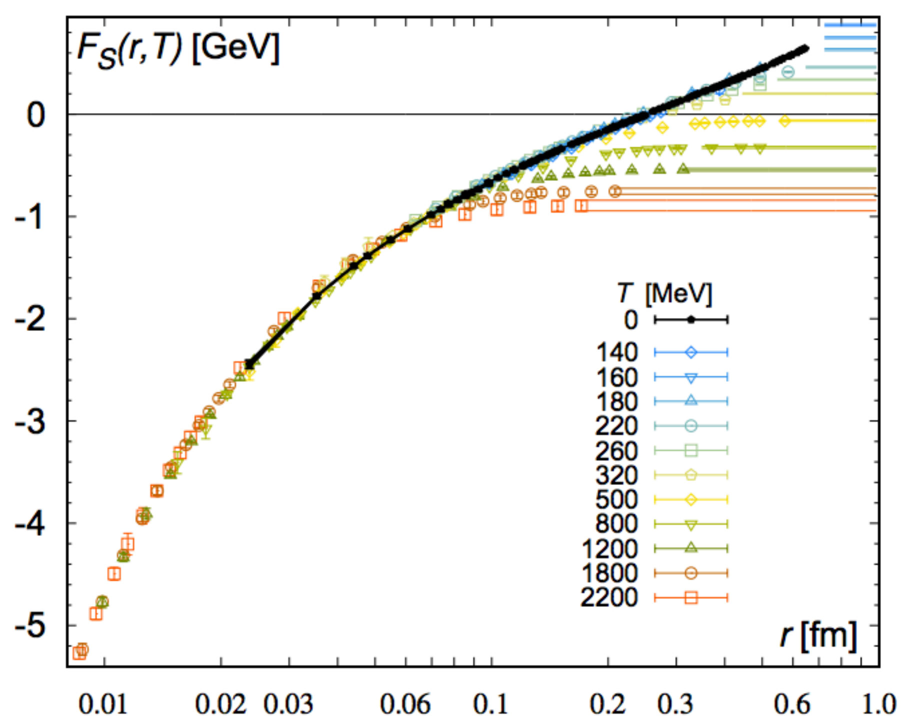
- **Quarkonium: screening**, dissociation and recombination



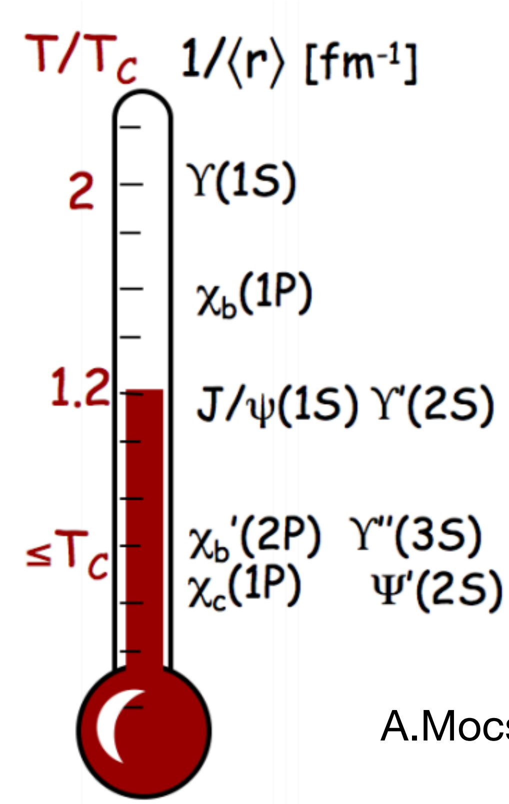
Ground and lower excited states described by Schrödinger equation

In vacuum: $V(r) = -\frac{A}{r} + Br \longrightarrow$ In QGP: $V(r) = -\frac{A}{r} e^{-m_D r}$

Screening: potential too weak to support bound state, melting of state, suppression



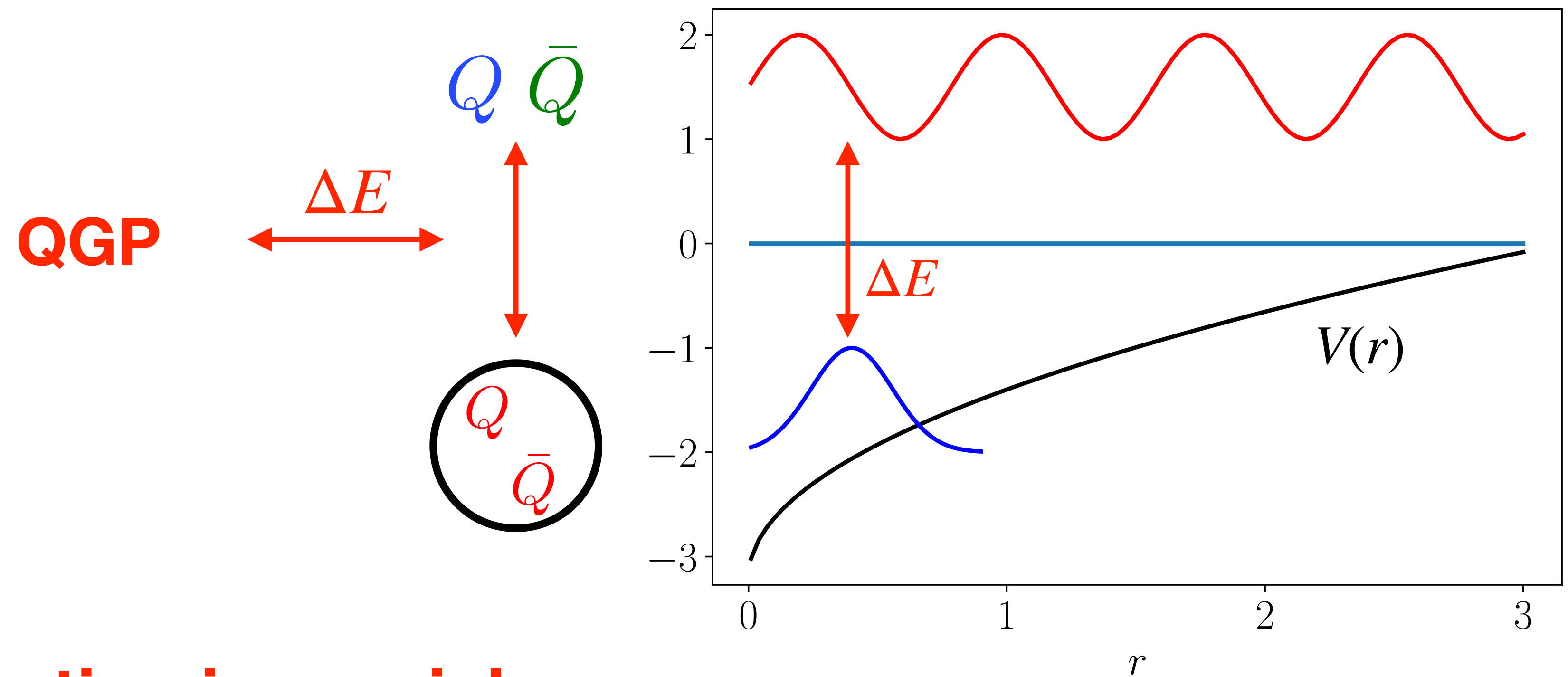
A.Bazavov, N.Brambilla, P.Petreczky, A.Vairo, J.H.Weber, 1804.10600



A.Mocsy, 0811.0337

Quarkonia inside QGP

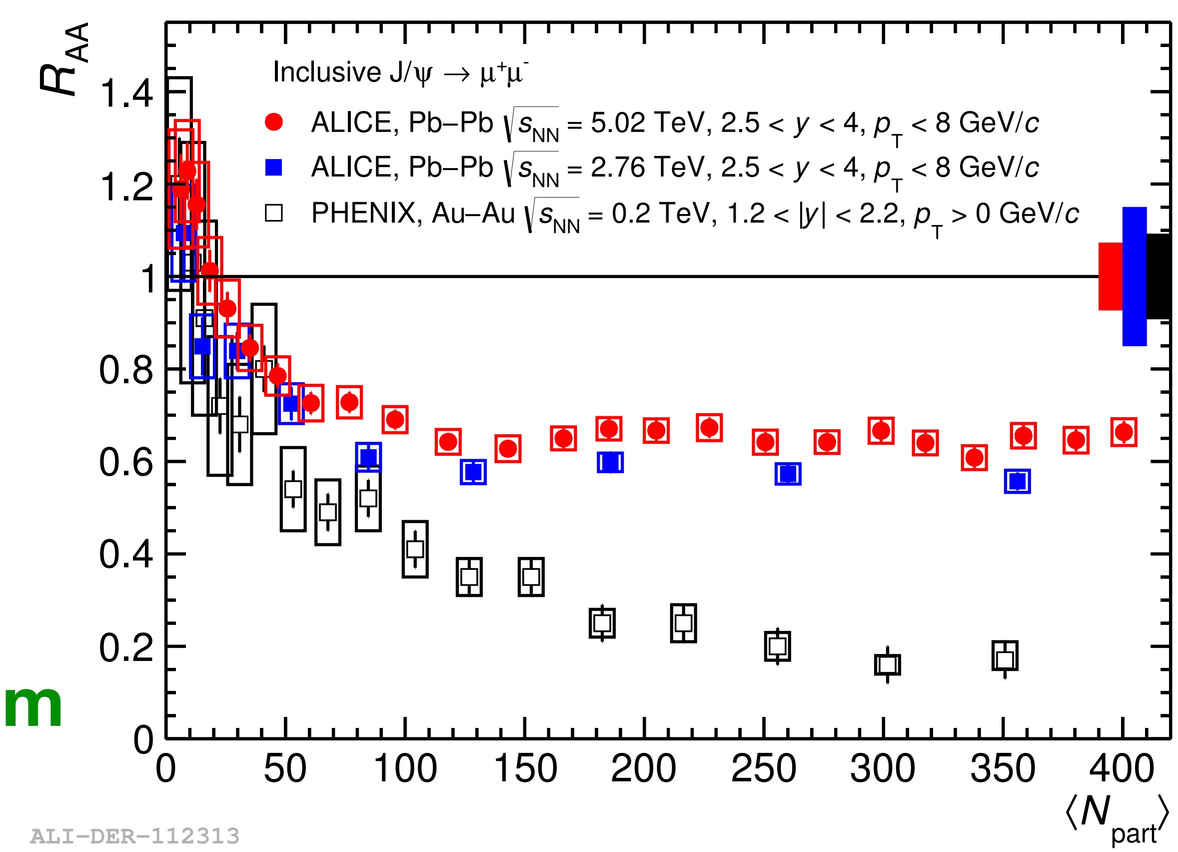
- **Quarkonium:** screening, **dissociation and recombination**



- **Recombination is crucial**

J/ψ less suppressed at higher energy: enhanced recombination from HQs produced from different hard collisions

Semiclassical transport equations model recombination, **go beyond semiclassical approach to understand recombination**



Treat heavy quarks as open quantum system

Open Quantum System

Total system = subsystem + environment: $H = H_S + H_E + H_I$

$\rho(t=0) = \rho_S \otimes \rho_E$

Subsystem & environment

(Heavy quarks & QGP)

Unitary evolution

Time reversible

$U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)$

Subsystem & environment

Trace out (integrate out) environment

$\rho_S(t=0)$

Subsystem

Non-unitary

Time irreversible

$\mathrm{Tr}_E\Big[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\Big]$

Review: XY, 2102.01736

Towards Lindblad Equation

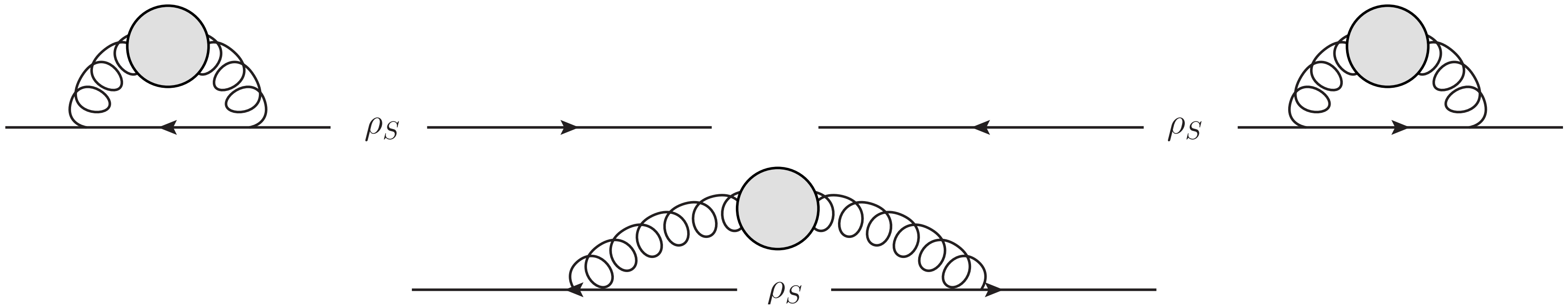
- Assume **weak coupling** between subsystem/environment

$$H = H_S + H_E + \boxed{H_I} \qquad H_I = \sum_\alpha O_\alpha^{(S)} \otimes O_\alpha^{(E)}$$

- Expand unitary evolution operator (time ordered perturbation theory)

- Trace out environment —> **finite-difference equation**

$$\rho_S(t) = \rho_S(0) - i\Big[tH_S + \sum_{a,b} \sigma_{ab}(t) L_{ab}, \rho_S(0)\Big] + \sum_{a,b,c,d} \gamma_{ab,cd}(t)\Big(L_{ab}\rho_S(0)L_{cd}^\dagger - \frac{1}{2}\{L_{cd}^\dagger L_{ab}, \rho_S(0)\}\Big)$$



$$\gamma_{ab,cd}(t) \equiv \sum_{\alpha,\beta} \int_0^t dt_1 \int_0^t dt_2 C_{\alpha\beta}(t_1, t_2) \langle a|O_\beta^{(S)}(t_2)|b\rangle \langle c|O_\alpha^{(S)}(t_1)|d\rangle^*$$

$$\sigma_{ab}(t) = \frac{-i}{2} \sum_{\alpha,\beta} \int_0^t dt_1 \int_0^t dt_2 \text{sign}(t_1 - t_2) \langle a|O_\alpha^{(S)}(t_1) O_\beta^{(S)}(t_2)|b\rangle C_{\alpha\beta}(t_1, t_2) \qquad L_{ab} = |a\rangle\langle b|$$

$$C_{\alpha\beta}(t_1, t_2) \equiv \text{Tr}_E(O_\alpha^{(E)}(t_1) O_\beta^{(E)}(t_2) \rho_E)$$

$|a\rangle$: Basis of Hilbert space in subsystem

Towards Lindblad Equation: Two Limits

Subsystem: non-unitary, time-irreversible evolution

$$\mathrm{Tr}_E\left[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\right]$$

Markovian (weak coupling)

Quantum optical limit (low T) → **Lindblad equation**

Quantum Brownian motion (high T) → **Lindblad equation**

$$\frac{d\rho_S(t)}{dt} = -i\left[H_{S,\mathrm{eff}}, \rho_S(t)\right] + \sum_n \left(L_n \rho_S(t) L_n^\dagger - \frac{1}{2}\{L_n^\dagger L_n, \rho_S(t)\}\right)$$

Wigner transform (smearing for positivity)

$$f(\boldsymbol{x},\boldsymbol{k},t) \equiv \int \frac{d^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} \left\langle \boldsymbol{k}+\frac{\boldsymbol{k}'}{2}\middle|\rho_S(t)\middle|\boldsymbol{k}-\frac{\boldsymbol{k}'}{2}\right\rangle$$

Semiclassical (gradient expansion)

Lindblad equation (low T) → **Boltzmann equation**

Lindblad equation (high T) → **Langevin/Fokker-Planck equation**

Hierarchy of Time Scales for Two Limits

- **Quantum optical limit (low T)**

$$\tau_R \gg \tau_E,\ \tau_R \gg \tau_S$$

- **Quantum Brownian motion (high T)**

$$\tau_R \gg \tau_E,\ \tau_S \gg \tau_E$$

τ_E: **environment correlation time**, time domain of environment correlator

$$C_{\alpha\beta}(t_1, t_2) \equiv \mathrm{Tr}_E(O_\alpha^{(E)}(t_1) O_\beta^{(E)}(t_2) \rho_E) \qquad \rho_E = \frac{e^{-\beta H_E}}{Z} \qquad \tau_E \sim \frac{1}{T}$$

τ_S: **subsystem intrinsic time**, inverse of typical energy gap

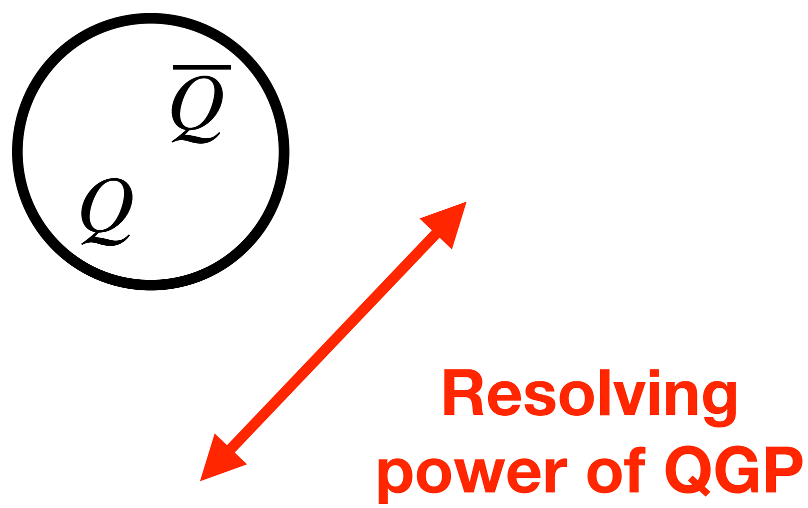
$$\tau_S \sim \frac{1}{\Delta H_S}$$

τ_R: **relaxation time**, depends on interaction strength between subsystem and environment

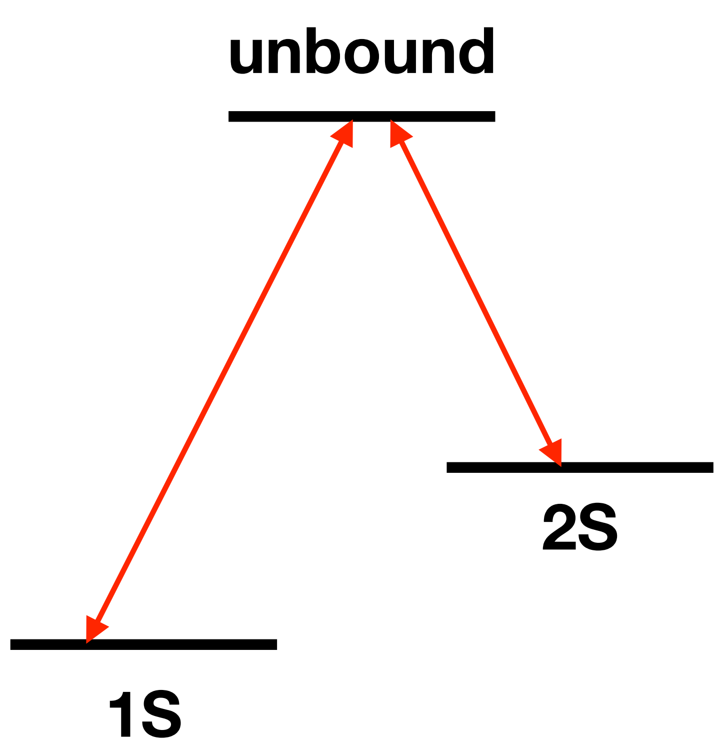
Physical Pictures of Two Limits

- **Quantum optical limit (low T)**

$$\tau_R \gg \tau_E,\ \tau_R \gg \tau_S$$

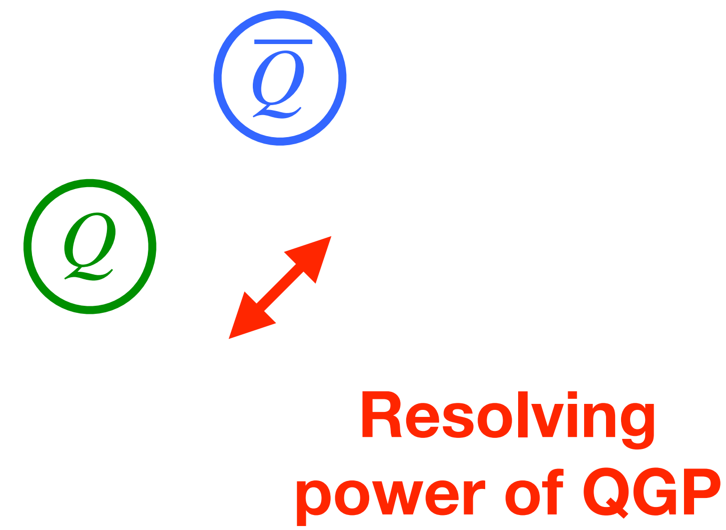


Transitions between levels

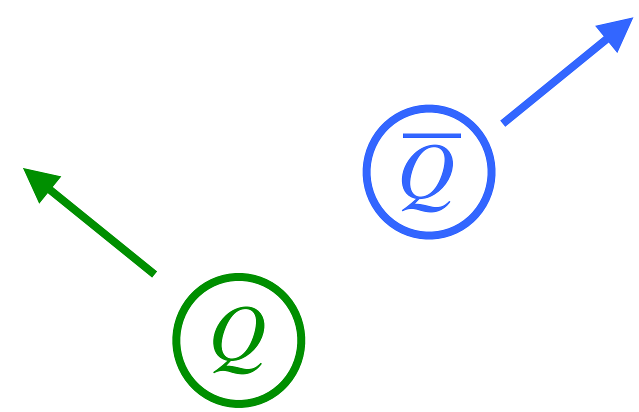


- **Quantum Brownian motion (high T)**

$$\tau_R \gg \tau_E,\ \tau_S \gg \tau_E$$



Diffusion of heavy Q pair



Wavefunction decoherence
—> dissociation

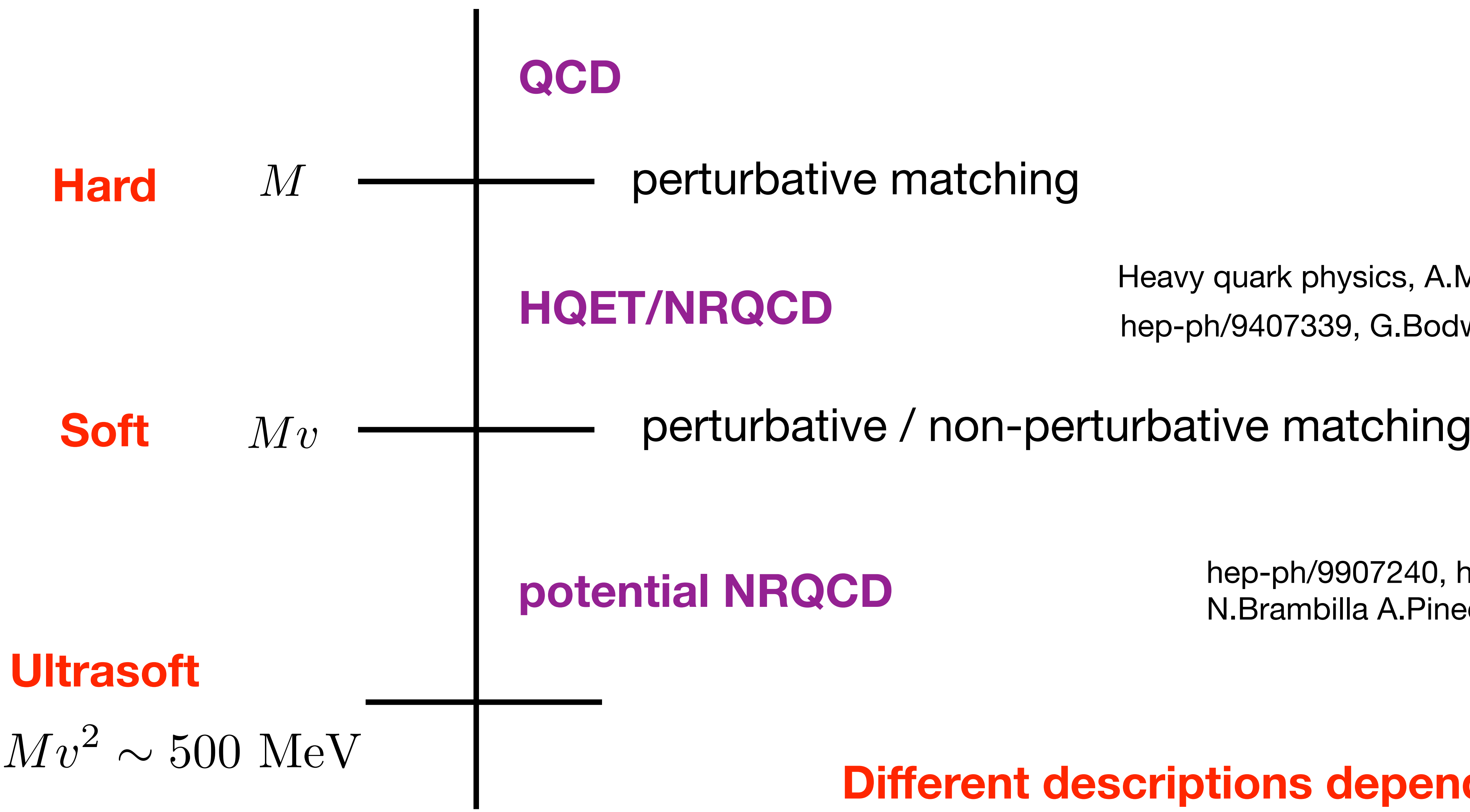
Separation of Scales and NREFT

- **Separation of scales**

$$M \gg Mv \gg Mv^2,\ \Lambda_{QCD}$$

$v^2 \sim 0.3$ charmonium

$v^2 \sim 0.1$ bottomonium



Heavy quark physics, A.Manohar, M.Wise

hep-ph/9407339, G.Bodwin, E.Braaten, G.Lepage

hep-ph/9907240, hep-ph/0410047,
N.Brambilla A.Pineda, J.Soto, A.Vairo

Different descriptions depending on where T fits into the hierarchy

High Temperature: NRQCD $M \gg T \gg Mv^2$

- **Lindblad equation in limit of quantum Brownian motion**

NRQCD motivated Hamiltonian

$$H = \frac{\hat{\boldsymbol{p}}_Q^2}{2M} + \frac{\hat{\boldsymbol{p}}_{\bar{Q}}^2}{2M} + H_{q+A} + \int d^3x \big(\delta^3(\boldsymbol{x} - \hat{\boldsymbol{x}}_Q) T_F^a - \delta^3(\boldsymbol{x} - \hat{\boldsymbol{x}}_{\bar{Q}}) T_F^{*a}\big) g A_0^a(\boldsymbol{x})$$

Lindblad equation

$$\frac{d\rho_S(t)}{dt} = -i\big[H_S + \Delta H_S, \rho_S(t)\big] + \frac{1}{N_c^2 - 1}\int \frac{d^3q}{(2\pi)^3} D^{>}(q_0 = 0, \boldsymbol{q})$$
$$\times \Big(\widetilde{O}^a(\boldsymbol{q})\rho_S(t)\widetilde{O}^{a\dagger}(\boldsymbol{q}) - \frac{1}{2}\big\{\widetilde{O}^{a\dagger}(\boldsymbol{q})\widetilde{O}^a(\boldsymbol{q}), \rho_S(t)\big\}\Big)$$

Zero frequency

Environment correlator $D^{>ab}(x_1, x_2) = g^2 \mathrm{Tr}_E\big(\rho_E A_0^a(t_1, \boldsymbol{x}_1) A_0^b(t_2, \boldsymbol{x}_2)\big)$

$$\widetilde{O}^a(\boldsymbol{q}) = e^{\frac{i}{2}\boldsymbol{q}\cdot\hat{\boldsymbol{x}}_Q}\Big(1 - \frac{\boldsymbol{q}\cdot\hat{\boldsymbol{p}}_Q}{4MT}\Big)e^{\frac{i}{2}\boldsymbol{q}\cdot\hat{\boldsymbol{x}}_Q} T_F^a - e^{\frac{i}{2}\boldsymbol{q}\cdot\hat{\boldsymbol{x}}_{\bar{Q}}}\Big(1 - \frac{\boldsymbol{q}\cdot\hat{\boldsymbol{p}}_{\bar{Q}}}{4MT}\Big)e^{\frac{i}{2}\boldsymbol{q}\cdot\hat{\boldsymbol{x}}_{\bar{Q}}} T_F^{*a}$$

Dissipation effect, important for thermalization, from expanding βH_S

J.-P. Blaizot, M.A.Escobedo, 1711.10812

T.Miura, Y.Akamatsu, M.Asakawa, A.Rothkopf, 1908.06293

N.Brambilla, M.A.Escobedo, M.Strickland, A.Vairo, P.V.Griend, J.H.Weber, 2012.01240

Solving this Lindblad equation expensive at 3D

Low Temperature: pNRQCD $Mv \gg Mv^2 \gtrsim T$

- **The potential NRQCD**

$$\mathcal{L}_{\mathrm{pNRQCD}} = \int d^3r\, \mathrm{Tr}\Big(\mathrm{S}^\dagger(i\partial_0 - H_s)\mathrm{S} + \mathrm{O}^\dagger(iD_0 - H_o)\mathrm{O} + \boxed{V_A(\mathrm{O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\mathrm{S} + \mathrm{h.c.})} + \frac{V_B}{2}\mathrm{O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, \mathrm{O}\} + \cdots\Big)$$

D.o.f. heavy quark pairs in color singlet (S) or octet (O)

Bound/unbound transition —> singlet/octet transition

Dipole interaction $r \sim \dfrac{1}{Mv}$

When at rest in medium, $rT \sim v$ suppressed

Weak coupling between quarkonium and QGP:
quarkonium small in size

At leading (nontrivial) order in v, sum all interactions not suppressed

Low Temperature: pNRQCD $Mv \gg Mv^2 \gtrsim T$

$$\rho_S(t) = \rho_S(0) - i\Big[tH_S + \sum_{a,b}\sigma_{ab}(t)L_{ab}, \rho_S(0)\Big] + \sum_{a,b,c,d}\gamma_{ab,cd}\Big(L_{ab}\rho_S(0)L^\dagger_{cd} - \frac{1}{2}\{L^\dagger_{cd}L_{ab}, \rho_S\}\Big)$$

Static screening

- **Boltzmann equation**

$$\frac{\partial}{\partial t} f_{nl}(\boldsymbol{x},\boldsymbol{k},t) + \frac{\boldsymbol{k}}{2M}\cdot\nabla_{\boldsymbol{x}} f_{nl}(\boldsymbol{x},\boldsymbol{k},t) = \mathcal{C}^+_{nl}(\boldsymbol{x},\boldsymbol{k},t) - \mathcal{C}^-_{nl}(\boldsymbol{x},\boldsymbol{k},t)$$

Recombination Dissociation

T.Mehen, XY, 1811.07027, 2009.02408

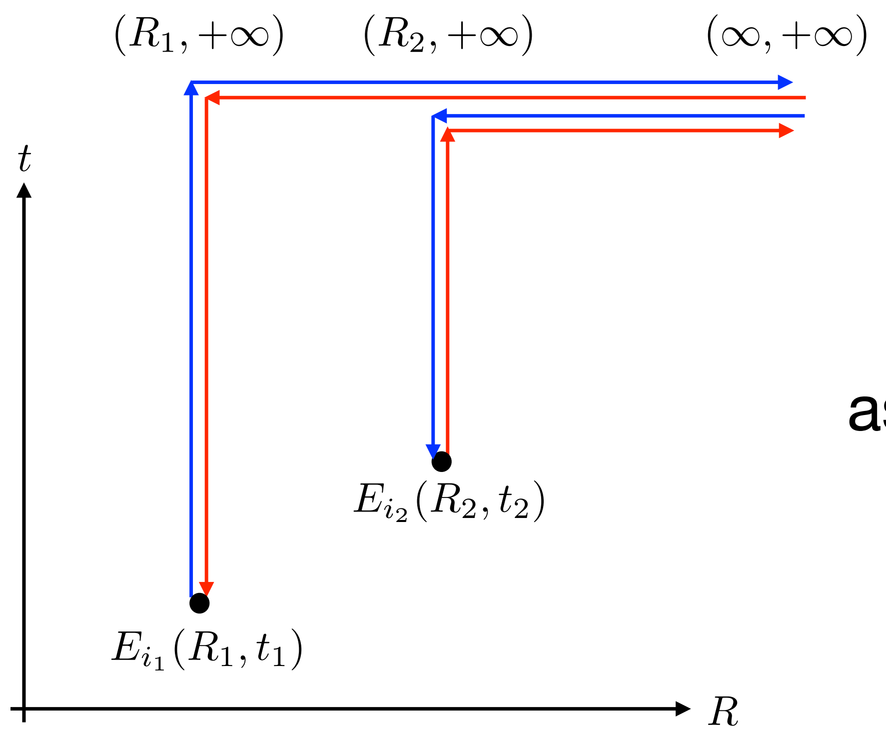
$$\mathcal{C}^-_{nl}(\boldsymbol{x},\boldsymbol{k},t) = g^2\frac{T_F}{N_c}\sum_{i_1,i_2}\int\frac{d^3p_{\rm cm}}{(2\pi)^3}\frac{d^3p_{\rm rel}}{(2\pi)^3}\frac{d^4q}{(2\pi)^4}(2\pi)^4\delta^3(\boldsymbol{k}-\boldsymbol{p}_{\rm cm}+\boldsymbol{q})\delta\Big(E_{nl}-\frac{p^2_{\rm rel}}{M}-q_0\Big)$$
$$\times\langle\psi_{nl}|r_{i_1}|\Psi_{\boldsymbol{p}_{\rm rel}}\rangle\langle\Psi_{\boldsymbol{p}_{\rm rel}}|r_{i_2}|\psi_{nl}\rangle\,[g_E^{++}]^{>}_{i_1i_2}(q_0,\boldsymbol{q})f_{nl}(\boldsymbol{x},\boldsymbol{k},t)$$

$$\mathcal{C}^+_{nl}(\boldsymbol{x},\boldsymbol{k},t) = g^2\frac{T_F}{N_c}\sum_{i_1,i_2}\int\frac{d^3p_{\rm cm}}{(2\pi)^3}\frac{d^3p_{\rm rel}}{(2\pi)^3}\frac{d^4q}{(2\pi)^4}(2\pi)^4\delta^3(\boldsymbol{k}-\boldsymbol{p}_{\rm cm}-\boldsymbol{q})\delta\Big(E_{nl}-\frac{p^2_{\rm rel}}{M}+q_0\Big)$$
$$\times\langle\psi_{nl}|r_{i_1}|\Psi_{\boldsymbol{p}_{\rm rel}}\rangle\langle\Psi_{\boldsymbol{p}_{\rm rel}}|r_{i_2}|\psi_{nl}\rangle\,[g_E^{--}]^{>}_{i_2i_1}(q_0,\boldsymbol{q})f_O(\boldsymbol{x},\boldsymbol{p}_{\rm cm},\boldsymbol{r}=0,\boldsymbol{p}_{\rm rel},t)$$

Chromoelectric Field Correlator

$$[g_E^{++}]^{>}_{ji}(y,x) \equiv \Big\langle \Big[E_j(y)\mathcal{W}_{[(y^0,\boldsymbol{y}),(+\infty,\boldsymbol{y})]}\mathcal{W}_{[(+\infty,\boldsymbol{y}),(+\infty,\boldsymbol{\infty})]}\Big]^a \times \Big[\mathcal{W}_{[(+\infty,\boldsymbol{\infty}),(+\infty,\boldsymbol{x})]}\mathcal{W}_{[(+\infty,\boldsymbol{x}),(x^0,\boldsymbol{x})]}E_i(x)\Big]^a\Big\rangle_T$$

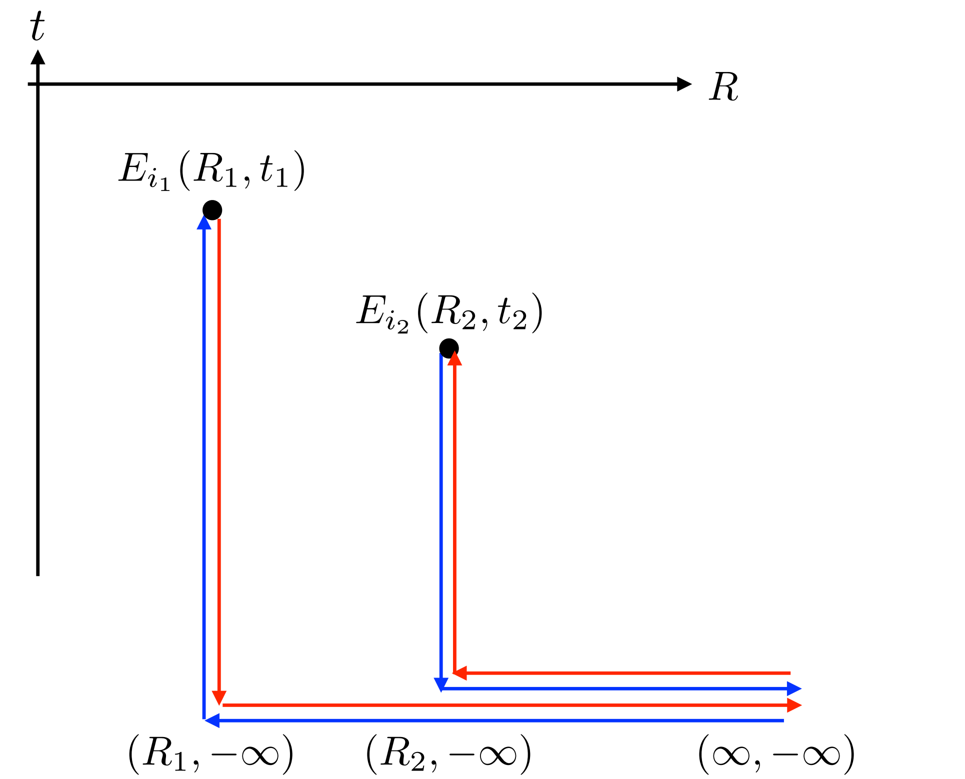
$$[g_E^{--}]^{>}_{ji}(y,x) \equiv \Big\langle \Big[\mathcal{W}_{[(-\infty,\boldsymbol{\infty}),(-\infty,\boldsymbol{y})]}\mathcal{W}_{[(-\infty,\boldsymbol{y}),(y^0,\boldsymbol{y})]}E_j(y)\Big]^a \times \Big[E_i(x)\mathcal{W}_{[(x^0,\boldsymbol{x}),(-\infty,\boldsymbol{x})]}\mathcal{W}_{[(-\infty,\boldsymbol{x}),(-\infty,\boldsymbol{\infty})]}\Big]^a\Big\rangle_T$$



PT transformation,
assume state invariant

⟷

KMS relation



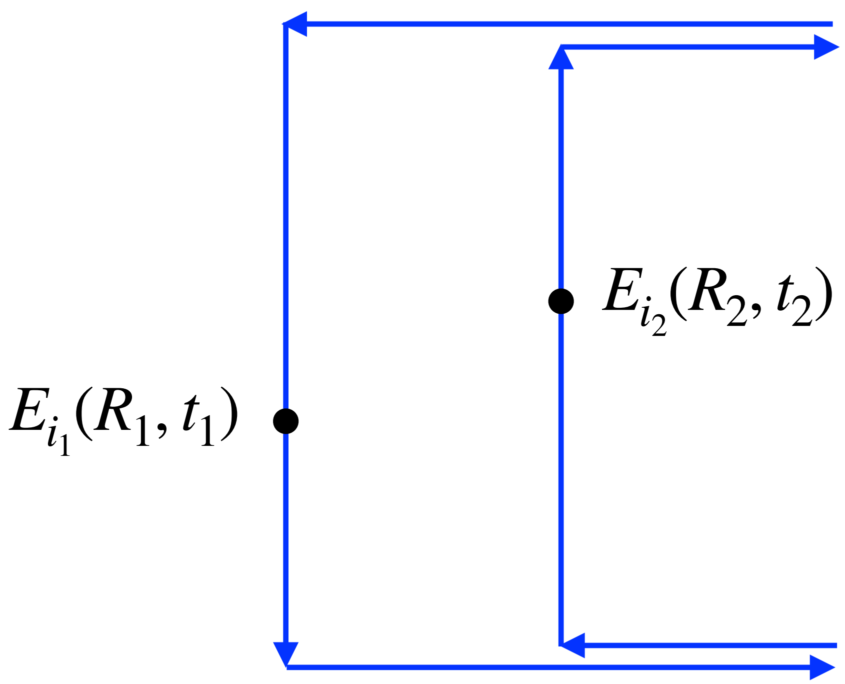
Dissociation: final-state interaction

Recombination: initial-state interaction

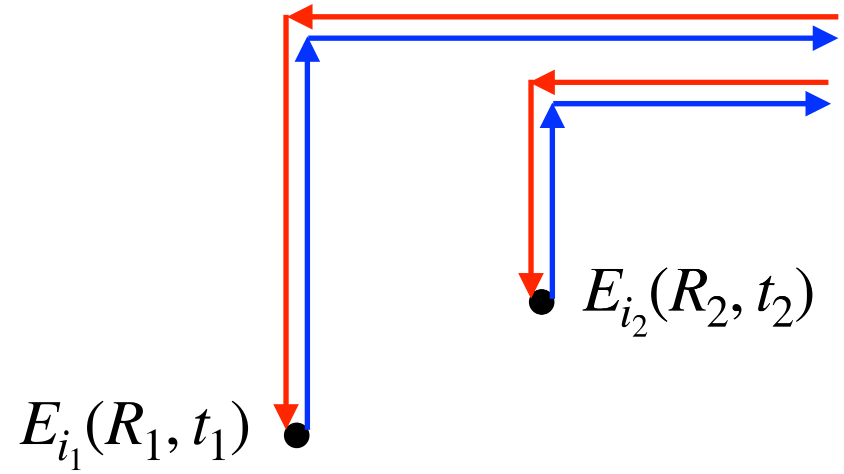
For total reaction rates, integrating over final momentum gives setting $R_1 \to R_2$, the correlator becomes momentum independent

Chromoelectric Field Correlator

- **Relation to the correlator defining heavy quark diffusion coefficient**



Single heavy quark



Heavy quark antiquark pair

- **At NLO: temperature-dependent parts of spectral functions agree**
  vacuum parts differ by a constant

$$\frac{149}{36} - \frac{2}{3}\pi^2$$

Y.Burnier, M.Laine, J.Langelage, L.Mether, 1006.0867

$$\frac{149}{36} + \frac{1}{3}\pi^2$$

T.Binder, K.Mukaida, B.Scheihing-Hitschfeld, XY, 2107.03945

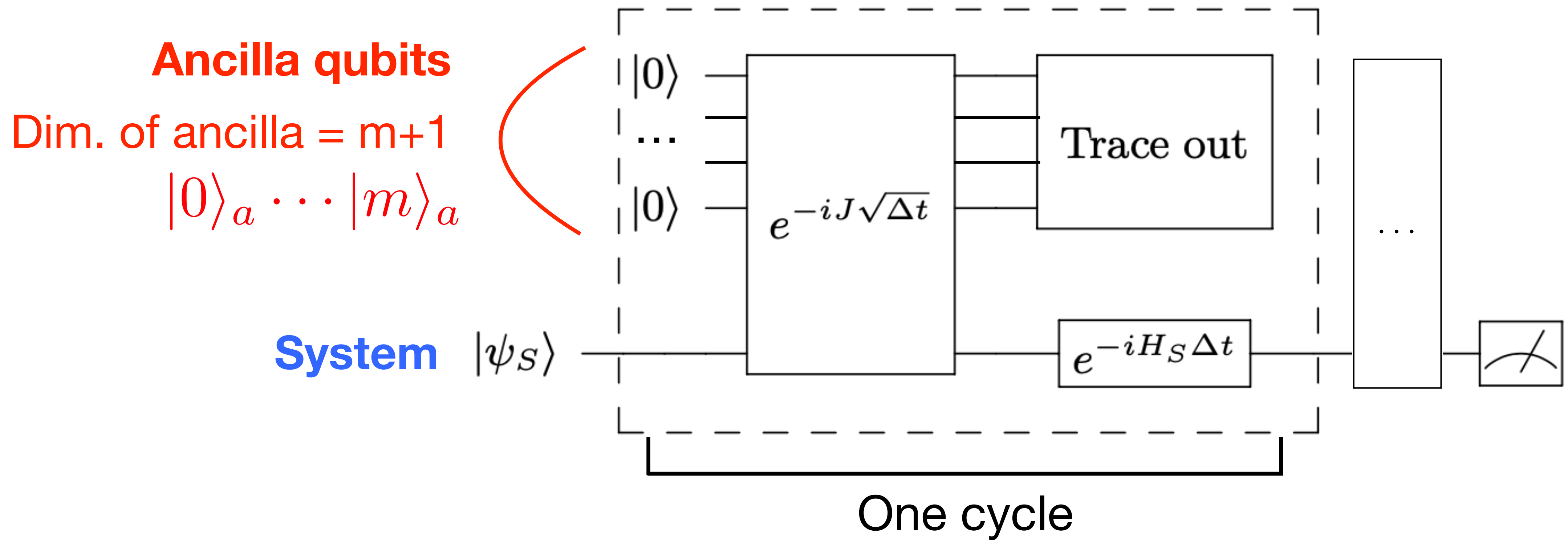
M.Eidemuller, M.Jamin, hep-ph/9709419 (only vacuum)

II. Quantum Simulation for Non-unitary Evolution of Open Quantum Systems

Stinespring Dilation Theorem

- **Completely positive trace-preserving map (e.g. Lindblad) = unitary evolution of system coupled with ancilla, and ancilla traced out**

$$\frac{d\rho_S(t)}{dt} = -i\left[H_S(t), \rho_S(t)\right] + \sum_{j=1}^{m} \left(L_j \rho_S(t) L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho_S(t)\}\right)$$



Reproduce Lindblad equation if expanded to Δt

$$J = \begin{pmatrix} 0 & L_1^\dagger & \dots & L_m^\dagger \\ L_1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ L_m & 0 & \dots & 0 \end{pmatrix}$$

$$\rho(0) = |0\rangle_a \langle 0|_a \otimes \rho_S(0) = \begin{pmatrix} \rho_S(0) & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}$$

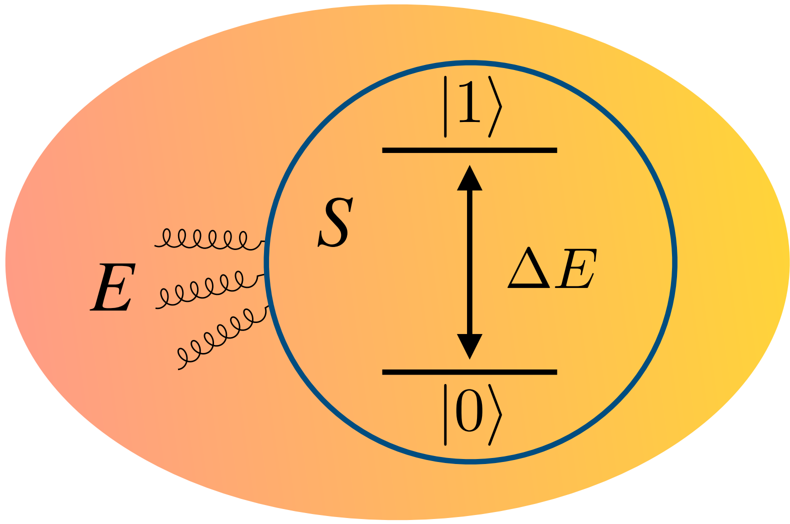
Two-Level System

- **Hamiltonian**

$$H_S = -\frac{\Delta E}{2} Z$$

$$H_E = \int d^3x \left[\frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4 \right]$$

$$H_I = gX \otimes \phi(x=0)$$



Resemble quarkonium dissociation and recombination

- **Quantum optical limit**

For $m = 0, \ \mathcal{O}(\lambda^0)$

$$L_1 = \sqrt{\frac{g^2 \Delta E n_B(\Delta E)}{8\pi}} (X - iY)$$

$$L_2 = \sqrt{\frac{g^2 \Delta E (1 + n_B(\Delta E))}{8\pi}} (X + iY)$$

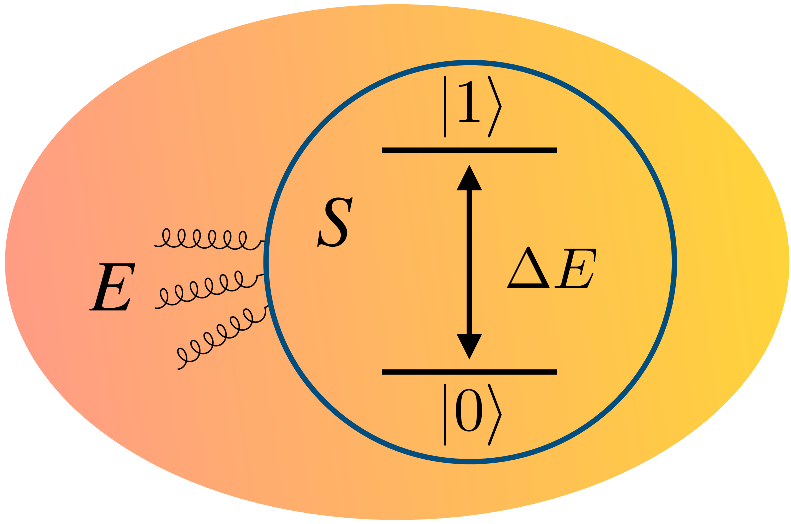
We need two qubits as ancilla, original environment has infinity degrees of freedom

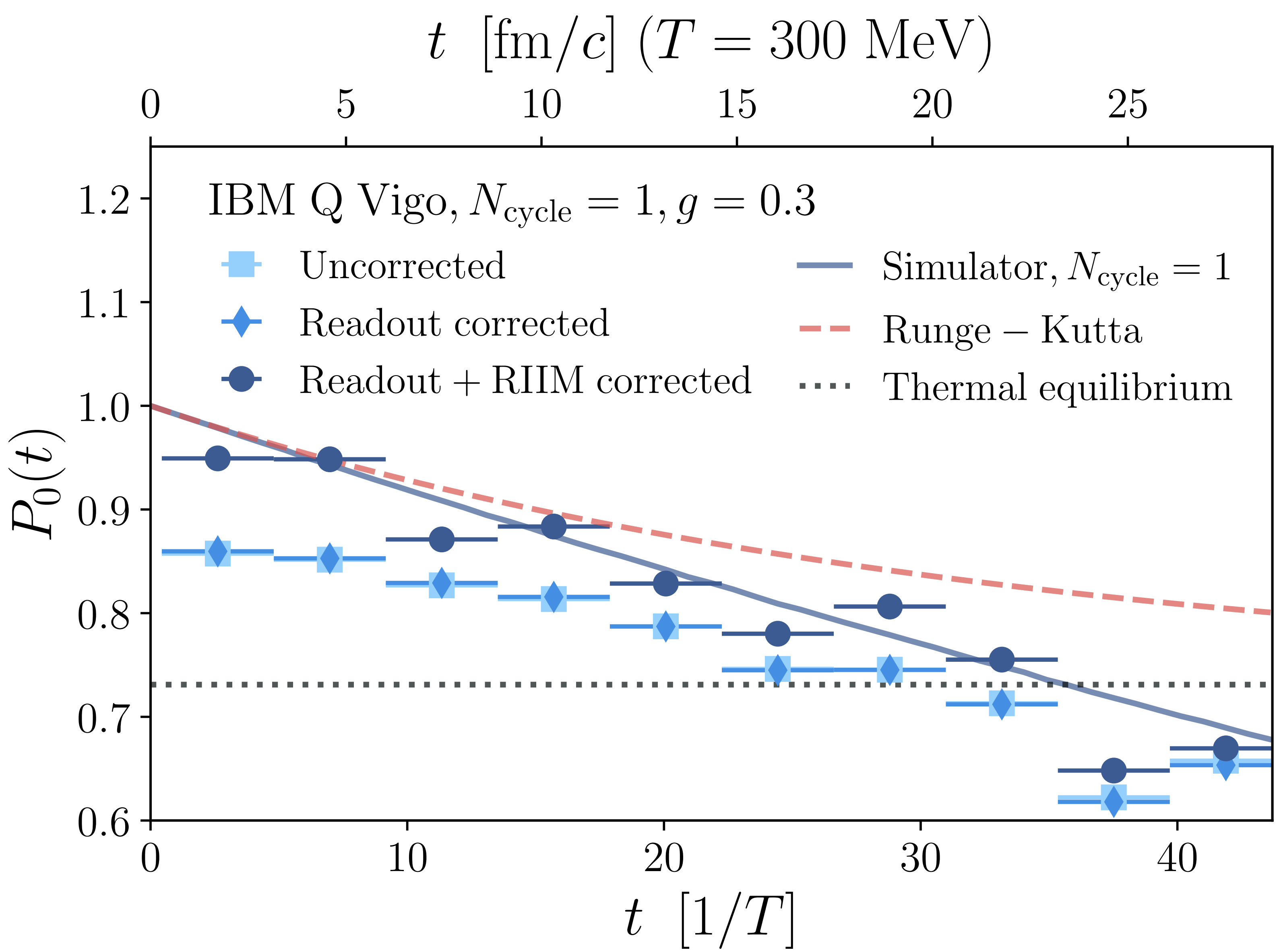
W.A.De Jong, M.Metcalf, J.Mulligan
M.Ploskon, F.Ringer and XY, 2010.03571

Quantum Simulation with Error Mitigation

- **Ground state probability**

$$P_0(t) = \langle 0|\rho_S(t)|0\rangle$$





IBM Q Vigo device

CNOT error mitigation crucial!

Schwinger Model

- **U(1) gauge theory in 1+1D**

$$\mathcal{L} = \overline{\psi}\big(iD^\mu\gamma_\mu - m\big)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

$$\gamma^0 = \sigma_z$$
$$\gamma^1 = -i\sigma_y$$

- **Hamiltonian formulation in axial gauge $A_0 = 0$**

$$\mathcal{H} = -i\overline{\psi}\gamma^1(\partial_1 + ieA)\psi + m\overline{\psi}\psi + \frac{1}{2}E^2$$

$$A = A_1$$
$$E = F^{10}$$

- **Discretization**

Staggered fermion $\chi(n) = \sqrt{a}\,(\gamma^1)^n\psi(n)$

J. B. Kogut and L. Susskind
Phys. Rev. D11(1975) 395–408

Jordan-Wigner transform $\chi(n) \to \Big(\prod_{m<n} i\sigma_z(m)\Big)\sigma^-(n)$

E field and gauge link solved by ladder $[E(n), U(n+1,n)] = eU(n+1,n)$

$E(n) \to \ell_n$, $U(n+1,n) \to L_n^+$

$$H_S = \frac{1}{2a}\sum_n \Big(\sigma^+(n)L_n^-\sigma^-(n+1) + \sigma^+(n)L_{n-1}^+\sigma^-(n-1)\Big) + \frac{m}{2}\sum_n(-1)^n\big(\sigma_z(n)+1\big) + \frac{ae^2}{2}\sum_n \ell_n^2$$

Schwinger Model Coupled w/ Thermal Scalars

- **Hamiltonians** $H = H_S + H_E + H_I$

$$H_E = \int dx \left[\frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{3!}g\phi^3\right]$$

$$H_I = \lambda \int dx\, \phi(x)\overline{\psi}(x)\psi(x) = \int dx\, O_E(x) O_S(x)$$

- **Lindblad equation in quantum Brownian motion limit**

$$\frac{d\rho_S(t)}{dt} = -i\left[H_S, \rho_S(t)\right] + L\rho_S(t)L^\dagger - \frac{1}{2}\left\{L^\dagger L, \rho_S(t)\right\}$$

Only one Lindblad operator: $L = \sqrt{aN_f D(k_0 = 0, k = 0)}\left(O_S - \frac{1}{4T}[H_S, O_S]\right)$

$$O_S^{\alpha\beta} = \frac{1}{aN_f}\sum_n \left\langle k = 0, \alpha \middle| \frac{(-1)^n(\sigma_z(n) + 1)}{2} \middle| k = 0, \beta \right\rangle$$

- **Observables**

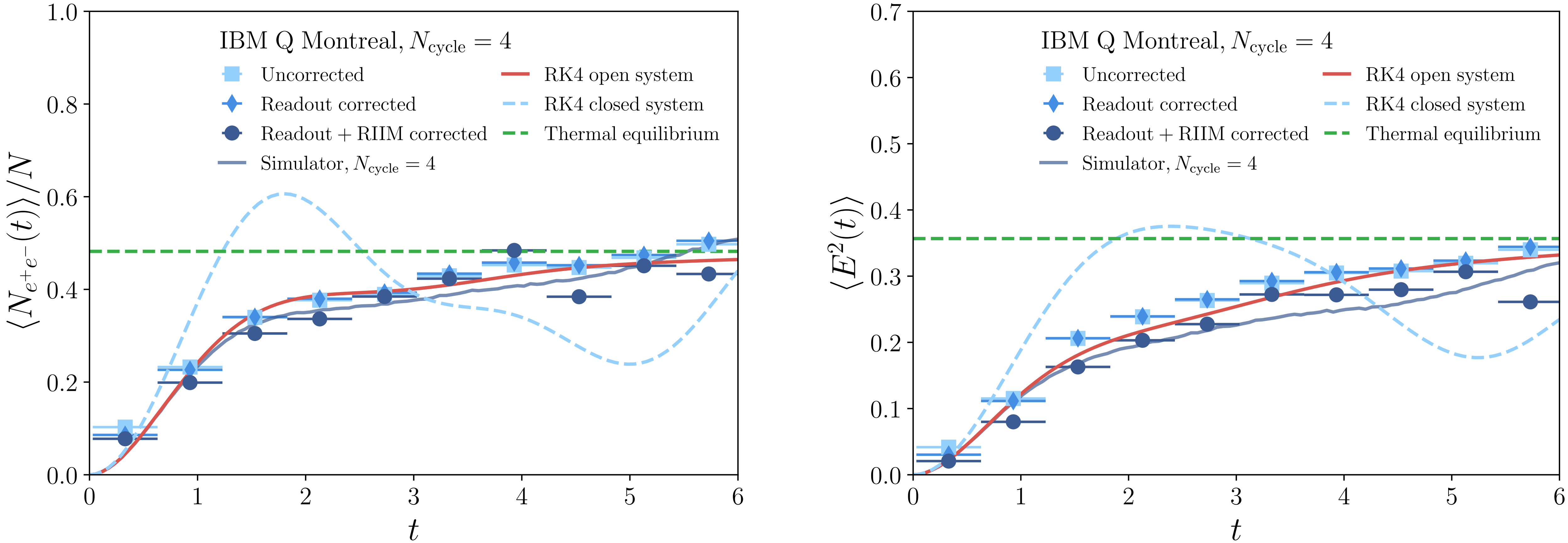
Average of E flux squared $\hat{A}_{E^2} = \frac{1}{2Na}\int dx\, E^2(x) = \frac{e^2}{2N}\sum_n \ell_n^2\,,$

Total number of fermion pairs $\hat{A}_{N_{e^+e^-}} = \sum_{n,\,\mathrm{even}} \sigma^+(n)\sigma^-(n)$

Results from Real Quantum Devices

$N = 2$ **spatial sites (4 fermion sites)**

$$e = \frac{1}{a},\ m = \frac{0.1}{a},\ \beta = 0.1a,\ a = 1$$



Possible to extend to higher number of cycles & compute observables close to thermal equilibrium

W.A.de Jong, K.Lee, J.Mulligan, M.Ploskon F.Ringer and XY, 2106.08394

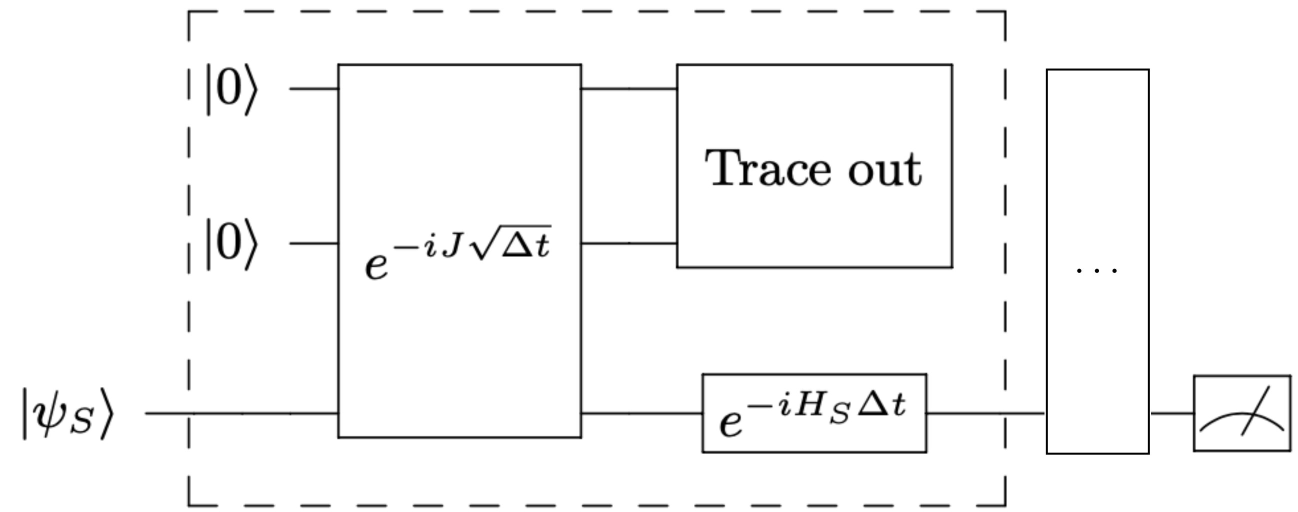
Conclusions

- Open quantum systems for heavy quarkonia in heavy ion collisions: derivation of Boltzmann equations in quantum optical limit, chromoelectric field correlators
- Quantum simulation of open quantum systems
- Future considerations:
  - Solving Lindblad equation in quantum Brownian motion by using quantum simulation (short depth)
  - Quantum simulation of non-Abelian gauge theories in higher dimensions

Backup: Quantum Circuit Synthesis

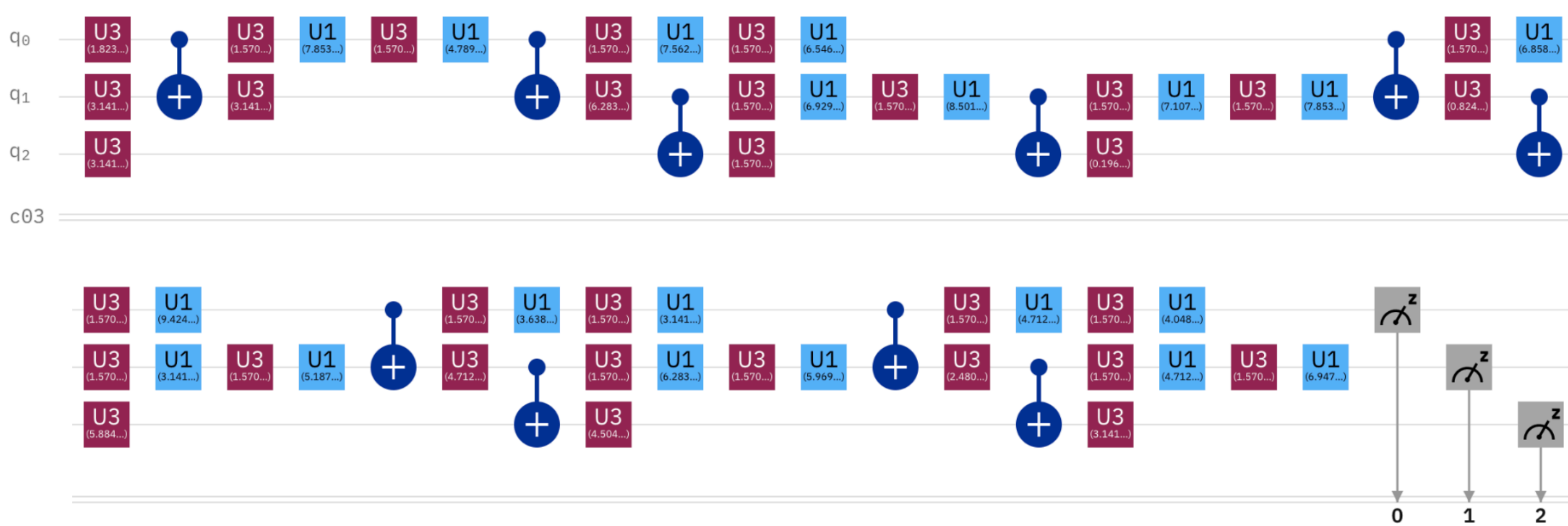
- **Compiling unitary evolution into single- and two-qubit gates**

Optimization based on **qsearch** package



~10 CNOT gates/cycle

IBM **Q**



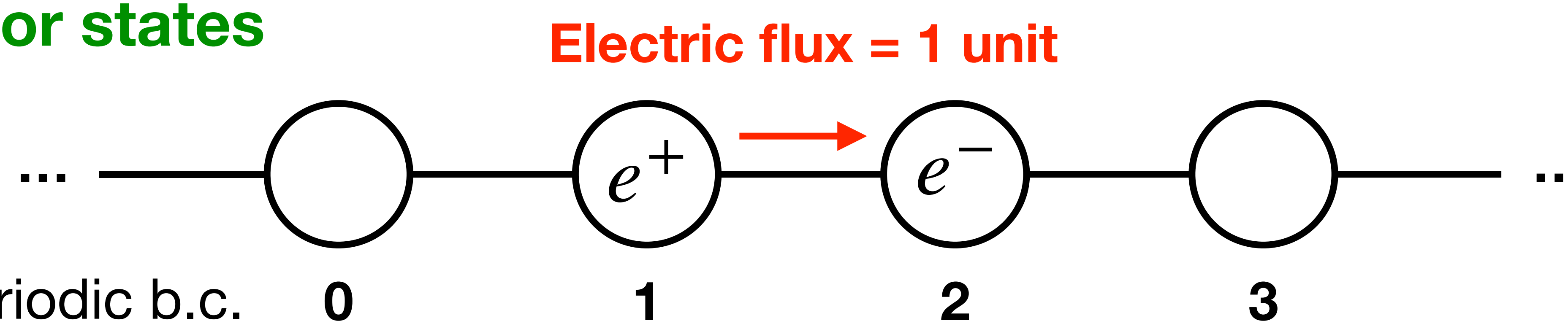
Single-qubit rotation 

CNOT 

Backup: States in Schwinger Model

- **Impose Gauss law for states**

$$\partial_1 E = e\psi^\dagger\psi$$



Even sites: fermion, odd sites: anti-fermion

- **Focus on states with specific momentum and symmetry**

k=0 state: first find states that are equivalent under cyclic permutation
then take symmetrized linear combination (trivial Fourier transform)

Positive parity: reflection w.r.t. a chosen site

- **Can write a code to generate the general states, specific momentum & parity states and corresponding H**

$$H_S^{\mathbf{k=0},+} =$$

$$\begin{pmatrix}
-4m & \frac{\sqrt{2}}{a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\sqrt{2}}{a} & \frac{ae^2}{2}-2m & \frac{1}{a} & \frac{1}{\sqrt{2}a} & \frac{1}{\sqrt{2}a} & \frac{1}{\sqrt{2}a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{a} & ae^2 & 0 & 0 & 0 & \frac{1}{2a} & \frac{1}{a} & \frac{1}{2a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}a} & 0 & ae^2 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}a} & 0 & 0 & ae^2 & 0 & 0 & \frac{1}{\sqrt{2}a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}a} & 0 & 0 & 0 & ae^2 & 0 & 0 & \frac{1}{\sqrt{2}a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2a} & 0 & 0 & 0 & \frac{3}{2}ae^2-2m & 0 & 0 & \frac{1}{2a} & \frac{1}{2a} & \frac{1}{2a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{a} & 0 & \frac{1}{\sqrt{2}a} & 0 & 0 & \frac{3}{2}ae^2+2m & 0 & \frac{1}{2a} & 0 & \frac{1}{2a} & \frac{1}{a} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2a} & \frac{1}{\sqrt{2}a} & 0 & \frac{1}{\sqrt{2}a} & 0 & 0 & \frac{3}{2}ae^2+2m & 0 & \frac{1}{2a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2a} & \frac{1}{2a} & 0 & 2ae^2 & 0 & 0 & 0 & \frac{1}{2a} & \frac{1}{2a} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2a} & 0 & \frac{1}{2a} & 0 & 2ae^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2a} & \frac{1}{2a} & 0 & 0 & 0 & 2ae^2 & 0 & \frac{1}{2a} & \frac{1}{2a} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a} & 0 & 0 & 0 & 0 & 2ae^2+4m & 0 & \frac{1}{a} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2a} & 0 & \frac{1}{2a} & 0 & \frac{5}{2}ae^2-2m & 0 & \frac{1}{2a} & & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2a} & 0 & \frac{1}{2a} & \frac{1}{a} & 0 & \frac{5}{2}ae^2+2m & \frac{1}{a} & \frac{1}{\sqrt{2}a} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2a} & \frac{1}{a} & 3ae^2 & 0 & \frac{1}{a} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}a} & 0 & 3ae^2 & \frac{1}{\sqrt{2}a} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a} & \frac{1}{\sqrt{2}a} & \frac{7}{2}ae^2-2m & \frac{1}{a} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a} & 4ae^2-4m
\end{pmatrix}$$

8 fermion sites

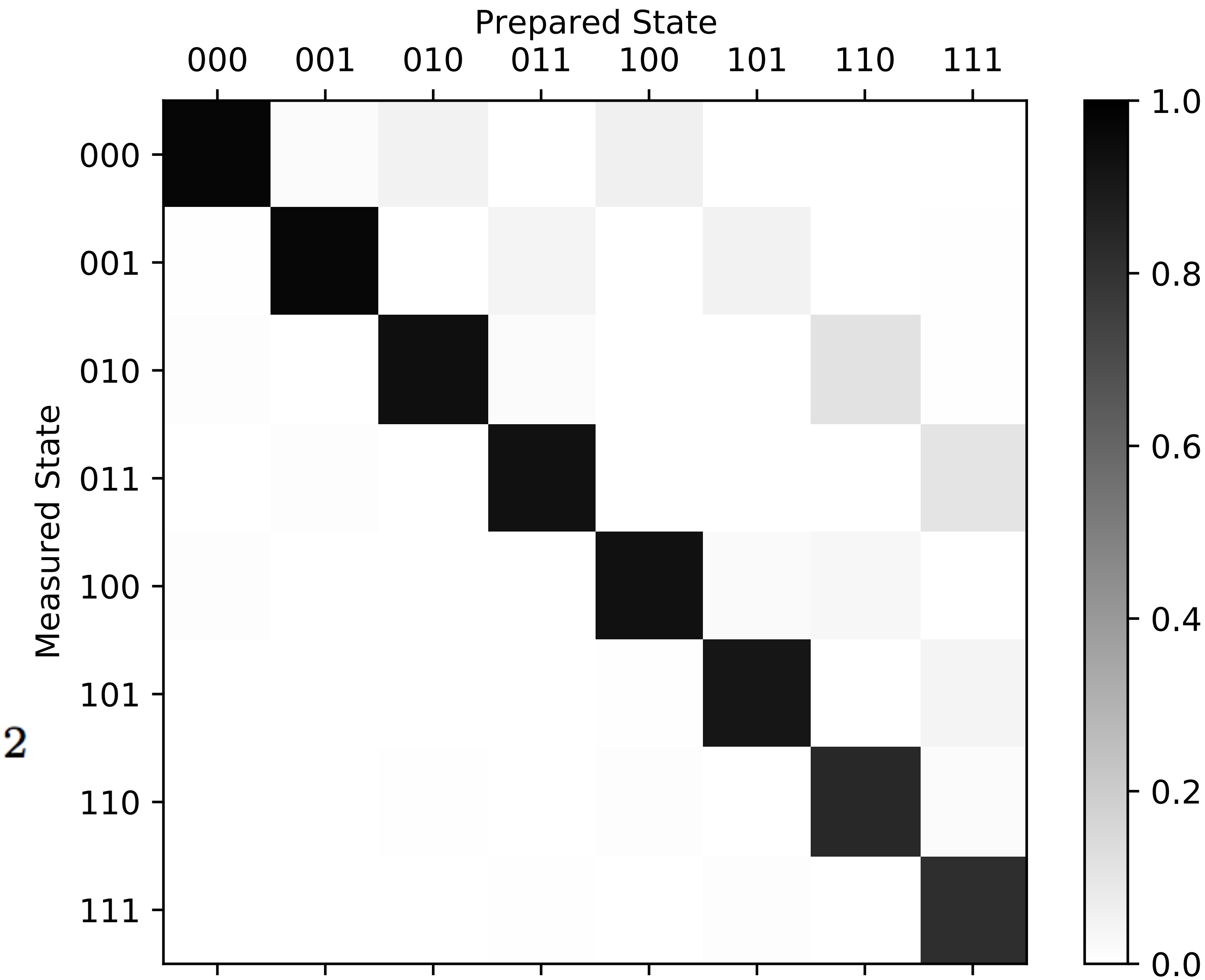
Backup: Readout Error Mitigation

$$m = Rt$$

$$t_{inverse} = R^{-1}m$$

ignis

$$\hat{t}_{\text{ignis}} = \underset{t':||t'||_1=||m||_1,\, t'_i>0}{\arg\min} ||m - Rt'||^2$$



Backup: Error Mitigation

- **Readout error**

  Constrained matrix inversion   qiskit-ignis package

- **Gate error**

  Zero-noise extrapolation of CNOT noise using Random Identity Insertions

He, Nachman, de Jong, Bauer 2003.04941